Intraday Lead-Lag Relationship between Stock Index and Stock Index Futures Markets: Evidence from Turkey

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\textbf{Abstract:} In perfectly frictionless and rational markets, spot markets and futures markets should simultaneously reflect new information. However, due to market imperfections, one of these markets may reflect information faster than the other and therefore may lead to the other. This study examines the lead-lag relationship between stock index and stock index futures, in terms of both price and volatility, by using 5 minute data over 2007-2010 period. The findings of this study indicate that a stable long-term relationship between Turkish stock index and stock index futures exists, however stock index futures do not lead stock index and there is a two way interaction between them. Therefore neither of the markets is dominant over the other one in the price formation process.

\textbf{Keywords:} Lead-Lag relationship, price discovery, volatility relationship.

\textbf{JEL Classification:} G13, G14, G15

1. Introduction

Price-risk protection requires a stable relationship between spot prices and futures prices. Large deviations from this relationship make it difficult to make optimal decisions regarding futures prices, increase the cost of risk protection for economic units and decrease efficiency in risk management. The complete breaking off of the relationship between spot and futures markets means completely independent movements of two markets, and in this case the use of futures markets in risk management and their price discovery role might prove impossible. Knowing how spot and futures markets are related will guide especially the transactions aimed at risk protection and aid all market participants in making rational decisions.

Theoretical foundations comprising the relationship between spot and futures markets are efficient markets hypothesis, cost of carry model (Hasan, 2005) and arbitrage. Fama (1970) defines an efficient market as the market where asset prices reflect all available information completely. Efficient markets hypothesis suggests that all available information will simultaneously be reflected both in spot prices and futures prices, and price movements in both markets be identically and independently distributed, resulting in efficient operating of financial markets. In an efficient capital market where interest rates and dividend yields are not stochastic, the main tenet of “cost of carry model” is a perfect relationship between simultaneous returns of futures and spot markets, hence no lead-lag relationship between them (Stoll and Whaley, 1990; Hasan, 2005). On the other hand, arbitrage steps in as a mechanism that brings it back to the cost of carry relationship both in price formation

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process and when the relationship between spot and futures markets to be established by the cost of carry model is disrupted.

In efficient and uninterrupted stock markets and futures markets where the interest rates and dividend yields are non-stochastic and no transaction costs and arbitrage opportunities exist, the cost of carry relationship must be valid at any time over the life of futures contract (Cornell and French, 1983; Stoll and Whaley, 1990). Under efficient market conditions where market imperfections do not exist, spot and futures market changes (returns) are simultaneously and perfectly related, and particularly one market would not lead another (Brooks, Rew and Ritson, 2001).

Spot prices and futures prices are different from each other due to the difference in the cost of carry (Chan, 1992). However, because in efficient markets prices will adapt to the new information precisely and simultaneously, there will be a simultaneously perfect relationship between the price changes of spot index and index futures contract. In other words, spot index and the price of index futures contract must simultaneously react to the new information in the markets and there should be no lead-lag relationship between the price changes in two markets. Nevertheless, some market imperfections can lead to a faster reaction of one market to information compared to the other one, and therefore can lead to lead-lag relationship between markets (Stoll and Whaley, 1990; Chan, 1992). Due to some characteristics such as low transaction costs and high leverage effect, futures markets are expected to react faster to new information in the market and to lead spot markets. Therefore price changes in futures markets are expected to be followed by price changes in spot markets.

When it reaches the market, both markets will react to the new information. But, what’s important is which market reacts faster to the new information. In other words, which market is informationally more efficient in the pricing process is what’s important (Floros and Vougas, 2007).

The purpose of this study is to investigate, by using intra-day data (5 minutes), whether there is a lead-lag relationship in terms of both price and volatility between Turkish index spot markets and index futures markets. In other words, whether or not one market leads the other one, hence in which market the information is reflected and prices are formed before the other one is investigated in this study. The number of studies on the lead-lag relationship between spot and futures markets in Turkey is limited, and these studies have used daily data. This study not only uses 5 minute data to investigate the intraday dynamic lead-lag relationship between markets, but also investigates the relationship between them based on both price and volatility formation. This is the key difference from previous studies through which the study is expected to contribute to the related literature.

The remainder of the study is organized as follows. Section 2 presents related literature while section 3 describes the methodology. The results of the analyses carried out to investigate price and volatility relations between spot and futures markets are presented and assessed in sections 4 and 5. The final section presents concluding remarks.

2. Review of Literature

Although there are studies which suggest long-term relationships between spot markets and futures markets and that spot markets lead futures markets or there are some studies suggesting a bilateral relationship between two markets, most studies find that futures markets lead spot markets.
In their study where they use intra-day data, Kawaller, Koch and Koch (1987), Harris (1989), Cheung and Ng (1990), Kutner and Sweeney (1991), Fleming, Ostdiek and Whaley (1996), Chu, Hsieh and Tse (1999) find that S&P 500 Index Futures lead S&P 500 Spot Index. Stoll and Whaley (1990) conclude that S&P 500 Index and MMI (Major Market Index) futures returns lead spot index returns. Chan (1992) finds an asymmetric lead-lag relationship between spot and futures markets for MMI and S&P 500 Index. He finds strong evidence that futures markets lead spot markets, and weak evidence that spot markets lead futures markets. Abhyankar (1998) examines the relationship between FTSE 100 Index and FTSE 100 Index Futures Contracts and based on linear causality analyses finds that futures contract returns lead returns on the spot index. On the other hand, based on non-linear causality analyses, he finds a strong two-way causality between spot market and futures market returns. In various studies by Brooks et al. (2001), Grünbichler, Longstaff and Schwartz (1994), Tse (1995), Floros and Vougas (2007), it is found that spot index lags behind FTSE 100 index futures contracts, German DAX index futures contracts, NSA (Nikkei Stock Average) index futures contracts and in Greece FTSE/ASE-20 and FTSE/ASE Mid 40 index futures contracts, respectively.

The following studies find two-way causality relationships between spot index and futures contracts for the country on which the study is based on: Wahab and Lashgari (1993) for FTSE 100 Index, Pizzi, Economopoulos and O'Neill (1998) for S&P 500 Index, Turkington and Walsh (1999) for SPI (Share Price Index) in Australia, Kenourgios (2004) for FTSE/ASE-20 Index in Greece, Hasan (2005) for FTSE 100 and S&P 500 Indexes. Pradhan and Bhat (2009) investigates the relationship between S&P CNX Nifty Index futures contracts and Nifty Spot Index and find that spot market leads futures market.

Kawaller, Koch and Koch (1990) examine the relationship between the volatility of S&P 500 Index and the price volatility of futures contracts based on the index. They find no systematic relationship by which futures market volatility leads spot market volatility (and vice versa). Chan, Chan and Karolyi (1991) find a strong relationship between S&P 500 and MMI indexes and the return volatilities of futures contracts on these indexes. Koutmos and Tucker (1996) conclude that the information on S&P 500 Index obtained from futures market can be used to predict the volatility in the spot market. Kang, Lee and Lee (2006), based on KOSPI 200 in Korea, find that the volatilities in the options and futures markets lead the spot market. Tse (1999) examines the volatility spillover between DJIA (Dow Jones Industrial Average) and DJIA Index futures contracts and finds a two-way significant information flow between the markets. Lafuente (2002) finds a two-way causality relationship between Ibex 25 and the volatilities of futures contracts on this index, in Spain.

Due to the fact that futures markets in Turkey (Turkish Derivatives Exchange-TurkDEX) have a very short history, there are limited number of studies on the subject, and except for Bekgoz (2006) which uses 5 minute data, these studies are based on daily data. Using data covering February 2005-December 2005 period, Bekgoz (2006) concludes that there is a two-way relationship between spot and futures markets, however the relationship from spot market to futures market is stronger. In other words the author concludes that spot market is a leading indicator of futures market. Based on July 2002-October 2007 data Kasman and Kasman (2008) find that the causality relationship in the long-run and short-run is from spot market to futures market. Examining the February 2005-September 2006 period, Sevil, Sayilir and Yalama (2008) find that spot market leads futures market. Dikmen (2008), and based on February 2005-May 2008 data Basdas (2009) find relations in the same way. For the period
January 2007-February 2009, Ozen, Bozdogan and Zugul (2009) conclude that the causality relationship between spot and futures markets in the long-run is bilateral, while in the short-run it is from futures market to spot market. Ozturk (2008) uses data covering January 2006-July 2008 period to examine the relationships of ISE-30 Index and ISE-100 Index with futures contracts on these indexes, with varying methods and concludes that the interaction between markets is from spot market to futures market. Using November 2005-September 2006 data, for ISE-100 Index Cevik and Pekkaya (2007) find that spot market leads futures market in terms of returns and there is a two-way feedback relationship in terms of return variances. The authors also find that the volatilities in the futures market and spot markets mutually affect each other on the same day. Kayalidere, Aracı and Aktas (2012) examine January 2006-December 2011 period in two sub-periods. They find causality relationships from spot market to futures market both for the full sample and pre-January 2009 period. Nevertheless, direction of the relationship is reversed in the post-January 2009 period. Celik (2012) uses data covering February 2005-February 2011 period and find causality relationship from spot market to futures market.

3. Data and Methodology

This study uses data covering the period January 2007-March 2010, resulting in 50,209 observations. It uses 5 minute data on TurkDEX-ISE 30 Futures Contracts (the most actively traded contract) and ISE-30 Index. 5 minute data used in the study were derived from tick-by-tick data obtained from ISE and TurkDEX.

Studies in the literature use data on the nearest futures contract in futures markets. In order to avoid expiration day effects, sometime before the nearest index futures contract expires, data on the next nearest futures contract are used. To avoid non-synchronized trading problem in studies using intraday data, time series are arranged in a manner to capture the time interval when the two markets are open at the same time. In addition, for the sake of avoiding the old information of the previous day, the first time interval observed for each day is ignored. In the event that no trading has occurred in a specific time interval, the data on the previous time interval are used as representative of that specific time interval (Stoll and Whaley, 1990; Chan, 1992; Abhyankar, 1998; Tse, 1999; Turkington and Walsh, 1999). In this study, we follow the same procedure.

The existence of a long-run steady relationship between ISE-30 index futures contracts and ISE-30 index is investigated via Johansen Cointegration Test. Lead-lag relationship between ISE-30 index futures and ISE-30 index is analyzed, by Granger Causality Test, for the purpose of decreasing the effect of micro-structural differences between the markets, using VAR model applied on the series obtained from Autoregressive Moving Average (ARMA) Model which is frequently used in the literature. On the other hand, the lead-lag relationship between the volatilities of the markets is analyzed by VAR and Granger Causality Test on the volatility series obtained from BEKK-MGARCH model.

Stationarity of the series need to be obtained before the cointegration relationship between spot market and futures market can be investigated. Towards this end, Augmented Dickey-Fuller (ADF) unit root test is applied on ISE-30 index and ISE-30 index futures series. Equation 1 and Equation 2 used in ADF unit root tests, include only the constant and both constant and deterministic trend, respectively;
\[
\Delta Y_t = \mu + \delta Y_{t-1} + \sum_{j=1}^{p} \delta_j \Delta Y_{t-j} + \varepsilon_t \tag{1}
\]

\[
\Delta Y_t = \mu + \beta_t + \delta Y_{t-1} + \sum_{j=1}^{p} \delta_j \Delta Y_{t-j} + \varepsilon_t \tag{2}
\]

In the regression equations used for ADF unit root test, \((\Delta Y_{t-j})\) stands for lagged values of the dependent variable used for avoiding serial correlation problem in the residuals, \(\Delta\) is the difference operator, \(\delta\) is the unknown autoregressive parameter, \(\mu\) is the constant term, \(\beta\) is the linear deterministic trend (linear time trend) and \(\varepsilon_t\) is the error term.

ADF unit root test is based on comparing the t-statistic, corresponding to \(\delta\) parameter estimated by Least Square (OLS) Estimator, with critical values. In ADF unit root test, the null hypothesis that unit root exists in the series and it’s non-stationary \((H_0; \delta = 0)\) is tested against the alternative hypothesis that unit root doesn’t exist in the series and it’s stationary \((H_1; \delta < 0)\) (Green, 2003). The t-statistic corresponding to the estimated \(\delta\) parameter is compared with MacKinnon’s (1996) critical values, and if t-statistic is above MacKinnon’s critical values, the null hypothesis can not be rejected, meaning that the series is non-stationary. Otherwise, the series is regarded as stationary.

Johansen (1991) cointegration test is used for determining whether long-run relationship exists between first difference stationary, ISE-30 index and ISE-30 index futures series. Johansen cointegration test is based on VAR model formulated using non-stationary time series. The test uses Vector Error Correction Model (VECM) developed by Johansen (1988, 1991) and Johansen and Juselius (1990). VECM model can be generated by using an autoregressive model \((\text{VAR}(p))\) with n-dimensions and p lags. Equation (3) represent a VAR model where \(X_t = (\text{Spot, Futures})\) and \(\varepsilon_t = (\varepsilon_{\text{spot}}, \varepsilon_{\text{futures}})\) and \(p\) is the lag length.

\[
X_t = A_0 + A_1 X_{t-1} + \cdots + A_p X_{t-p} + \varepsilon_t \tag{3}
\]

In this equation \(\varepsilon_t\) is an identically and independently distributed vector error series which has a normal distribution with zero mean and constant variance. For the sake of employing Johansen cointegration test, VAR(p) process in equation (3) can be converted into a vector error correction model (VECM) as shown by equation (4) (Brooks, 2006);

\[
\Delta X_t = \Pi X_{t-1} + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \cdots + \Gamma_{p-1} \Delta X_{t-(p-1)} + \varepsilon_t \tag{4}
\]

In the VECM model shown by equation (4), \(\Pi = \sum_{l=1}^{p} a_l - l_g\) and \(\Gamma = \left(\sum_{l}^{q} q_l \right) - l_g\). \(A_0\) stands for \((n x 1)\) vector of intercepts \((a_01, a_02 \ldots a_{0n})\), \(X_t\) and \(\varepsilon_t\) stand for \((n x 1)\) vector, \(A_1\) stands for \((n x n)\) parameter matrix, \(l\) stands for \((n x n)\) unit matrix and \(\Pi\) stands for \((A_1 - l_g)\). The rank of \((A_1 - l_g)\) value is equal to the number of cointegrating vectors. For example, when this value is equal to zero \([(A_1 - l_g) = 0]\), rank \((\Pi = 0)\), which will be an indication of no cointegration relationship between variables (Kutlar, 2000). Johansen cointegration test focuses on examining the long-term coefficient matrix \(\Pi\). \(r\) is the rank of \(\Pi\) matrix and shows whether the system is co-integrated. If \(r = 0\), the series are not cointegrated (Brooks, 2006).
For investigating the cointegration relationship between the series under the Johansen approach, trace ($\lambda_{trace}$) and maximum eigenvalue ($\lambda_{max}$) test statistics are used;

$$\lambda_{trace} (r) = -T \sum_{i=r+1}^{n} \ln(1 - \hat{\lambda}_i)$$ \hfill (5)

$$\lambda_{max} (r, r+1) = -T \ln(1 - \hat{\lambda}_{r+1})$$ \hfill (6)

In equations (5) and (6), where $r$ is the number of cointegration vectors under the null hypothesis, $\hat{\lambda}_i$ is the estimated value for the $i_{th}$ ordered eigenvalue from the $\Pi$ matrix and $T$ is the usable number of observations. Trace test is used for testing the null hypothesis that at most $r$ cointegrating vectors exist against the alternative hypothesis that more than $r$ cointegrating vectors exist. Maximum eigenvalue test has its null hypothesis that the number of cointegrating vectors is $r$ against an alternative hypothesis of $r + 1$ cointegrating vectors. If the test statistics $\lambda_{trace}$ or $\lambda_{max}$ are greater than the critical values for certain significance levels, then the null hypothesis is rejected against the alternative hypothesis, and series are regarded as cointegrated (Enders, 1995; Brooks, 2006).

The lead-lag relationship between ISE 30 index and ISE-30 index futures series is investigated by using Granger causality test. VAR models popularized in econometrics by Sims (1980) are regression systems where there are more than one dependent variable. VAR models do not require to make a distinction between exogenous (independent) and endogenous (dependent) variables (Brooks, 2006). Whenever one is not sure whether a variable is in fact exogenous (independent) a VAR model can be used. In the event of two variables, considering that the values of $Y_t$ over time are affected by both the current and past values of $Z_t$ variable, while the values of $Z_t$ over time are affected by both the current and past values of $Y_t$ variable, the standard form of a two-variable VAR model can be depicted by the following equations;

$$Y_t = a_{10} - a_{11}Y_{t-1} + a_{12}Z_{t-1} + e_{1t}$$ \hfill (7)

$$Z_t = a_{20} - a_{21}Y_{t-1} + a_{22}Z_{t-1} + e_{2t}$$ \hfill (8)

Variable $a_{11}$ ($a_{22}$) in equation (7) and equation (8) represents the effects of changes in lagged values of $Y_t$ ($Z_t$) dependent variable on the variable itself. On the other hand, $a_{12}$ represents the effects of changes in lagged values of $Z_t$ on $Y_t$, while $a_{21}$ represents the effects of changes in lagged values of $Y_t$ on $Z_t$. and $e_{1t}$ and $e_{2t}$ represent error terms (Enders, 1995).

The presence of explanatory power relation between two stationary time series and the direction of the explanatory power is based on Granger (1969) causality test. If variable $x_t$ is the Granger cause of variable $y_t$, changes in $x_t$ should precede changes in $y_t$. If adding $x_t$ or its lagged values to the regression of $y_t$ on other explanatory variables (including its own lagged values) improves the forecast of $y_t$ significantly, then $x_t$ is said to Granger cause $y_t$ (Gujarati, 2006). One of the good features of VAR models is that they allow us to test for the direction of causality. When one needs to investigate whether $y_t$ causes $x_t$ or $x_t$ causes $y_t$, or the causality between them is bi-directional or finally they are independent, a VAR model can be used to capture the relationship between these variables. Granger causality between two time series variables $y_t$ and $x_t$ can be tested by estimating the following VAR model.
This study examines the price relationship between spot and futures markets as well as the volatility relationship between them. Volatility series need to be generated for investigating the relationship between volatilities of ISE-30 index and ISE-30 index futures contracts. Therefore first, volatilities of the variables are modelled using BEKK-MGARCH (1,1) and then volatility series of the variables are obtained from the model. The relationship between volatility series obtained from BEKK-MGARCH (1,1) are examined by VAR and Granger Causality test.

In the event of a single variable in modelling asset price volatility ARCH, GARCH models are used, whereas multivariate GARCH (M-GARCH) models are used for measuring the common structure in case of more than one variable (Erdoğan and Bozkurt, 2009). MGARCH models are most widely used in studies investigating the relations between the volatilities and co-volatilities of several markets (Bauwens, Laurent and Rombouts, 2006). Bollerslev, Engle and Wooldridge (1988) expanded univariate ARCH models to multivariate models and reached solutions through vectorization parameterization. Afterwards, following Baba, Engle, Kraft and Kroner, BEKK parameterization where conditional variance matrix is guaranteed to be positively defined was developed by Engle and Kroner (1995). Proving whether conditional variances of multivariate GARCH models satisfy positive definiteness condition is generally difficult. BEKK (Baba, Engle, Kraft and Kroner) model suggested by Engle and Kroner (1995) satisfies this condition by its nature. In this study, having p as the lag length Vector Auto Regressive (VAR) model with two variables can be shown as the following for and (logarithmic return series);

\[
y_t = a_1 + \sum_{i=1}^{n} \beta_i x_{t-i} + \sum_{j=1}^{m} \gamma_j y_{t-j} + e_{1t} \tag{9}
\]

\[
x_t = a_2 + \sum_{i=1}^{n} \theta_i x_{t-i} + \sum_{j=1}^{m} \delta_j y_{t-j} + e_{2t} \tag{10}
\]

In equations (9) and (10) m and n are lagged values of \(y_t\) and \(x_t\), and \(e_{1t}\) and \(e_{2t}\) are uncorrelated white-noise error terms (Asteriou and Hall, 2007). VAR models indicate that independent variable is affected by its own lagged values and the lagged values of the other variables. Therefore, the null hypothesis (\(x_t\) does not Granger cause \(y_t\)) and alternative hypothesis (\(x_t\) Granger cause \(y_t\)) in the Granger causality test based on the VAR model is

\[
H_0: \beta_1 = \beta_2 = \beta_3 = \ldots = \beta_n = 0 \quad H_0: \theta_1 = \theta_2 = \theta_3 = \ldots = \theta_m = 0
\]

\[
H_1: \beta_1 = \beta_2 = \beta_3 = \ldots = \beta_n \neq 0 \quad H_1: \theta_1 = \theta_2 = \theta_3 = \ldots = \theta_m \neq 0
\]

Equations (9) and (10) covers 4 different cases: i) \(x_t\) causes \(y_t\), if the lagged \(x\) terms in (9) are statistically different from zero as a group, and the lagged \(y\) terms in (10) are not statistically different from zero. ii) \(y_t\) causes \(x_t\), if the lagged \(y\) terms in (10) are statistically different from zero as a group, and the lagged \(x\) terms in (9) are not statistically different from zero. iii) There is a bi-directionally causality between the variables, if both sets of \(x\) and \(y\) terms are statistically different from zero in (9) and (10). iv) The variables are independent of each other, if both sets of \(x\) and \(y\) terms are not statistically different from zero in (9) and (10) (Asteriou and Hall, 2007).

This study examines the price relationship between spot and futures markets as well as the volatility relationship between them. Volatility series need to be generated for investigating the relationship between volatilities of ISE-30 index and ISE-30 index futures contracts. Therefore first, volatilities of the variables are modelled using BEKK-MGARCH (1,1) and then volatility series of the variables are obtained from the model. The relationship between volatility series obtained from BEKK-MGARCH (1,1) are examined by VAR and Granger Causality test.
In $\text{VAR}(p)$ equation let $\varepsilon_t = \begin{bmatrix} \varepsilon_{\text{dlnspot} \ t}, \varepsilon_{\text{dlnfutures} \ t} \end{bmatrix}$ be a vector error process with two variables which is assumed to have a zero-mean and non-singular $\sum_{\varepsilon}$ covariance matrix. In this case conditional variance of residuals is denoted by $\text{Var}(\varepsilon_t | \mathcal{H}_t) = H_t$. $H_t$ is a conditional variance-covariance matrix with a dimension of $N \times N$. Diagonal elements of $H_t$ matrix denote variance terms while the other elements denote common variance terms. Vector error process $\varepsilon_t$ has a normal distribution with a zero matrix of dimension 2x1 and a covariance matrix $H_t$. Variance-covariance matrix and $H_t$ is in the form that $\text{vech}(H_t) = h_{\text{dlnspot} \ t,\text{dlnfutures} \ t}, h_{\text{dlnfutures} \ t,\text{dlnspot} \ t}, \varepsilon_{\text{dlnspot} \ t}, \varepsilon_{\text{dlnfutures} \ t}$'s. In other words $(\varepsilon_t | \Omega_{t-1}) \sim N(0, H_{t-1})$ and $\Omega_{t-1}$ is the information set until the period $t-1$. Following Engle and Kroner (1995) in this study, a $\text{VAR}(p)$-BEKK MGARCH specification is used and Granger causality test is performed based on the assumption that conditional matrices follow BEKK model. Parametric expression of variance-covariance matrix $H_t$ is as in equation (12):

$$H_t = C' C + A' \varepsilon_{t-1} \varepsilon_{t-1}' A + B' H_{t-1} B$$

Terms $C$, $A$ and $B$ are as the following, respectively:

$$C = \begin{bmatrix} C_{\text{dlnspot} \ t, \text{dlnspot} \ t} & C_{\text{dlnspot} \ t, \text{dlnfutures} \ t} \\ C_{\text{dlnfutures} \ t, \text{dlnspot} \ t} & C_{\text{dlnfutures} \ t, \text{dlnfutures} \ t} \end{bmatrix}, \quad A = \begin{bmatrix} A_{\text{dlnspot} \ t, \text{dlnspot} \ t} & A_{\text{dlnspot} \ t, \text{dlnfutures} \ t} \\ A_{\text{dlnfutures} \ t, \text{dlnspot} \ t} & A_{\text{dlnfutures} \ t, \text{dlnfutures} \ t} \end{bmatrix}$$

$$B = \begin{bmatrix} B_{\text{dlnspot} \ t, \text{dlnspot} \ t} & B_{\text{dlnspot} \ t, \text{dlnfutures} \ t} \\ B_{\text{dlnfutures} \ t, \text{dlnspot} \ t} & B_{\text{dlnfutures} \ t, \text{dlnfutures} \ t} \end{bmatrix}.$$

In case that all diagonal elements of matrix $C$ are positive and $a_{\text{dlnspot} \ t, \text{dlnspot} \ t}, b_{\text{dlnspot} \ t, \text{dlnspot} \ t} > 0$, it is accepted that BEKK-MGARCH(1,1) type models denoted by equation (1) are estimated correctly. $a_{\text{dlnspot} \ t, \text{dlnspot} \ t}$ and $b_{\text{dlnspot} \ t, \text{dlnspot} \ t}$ are top left corner elements of matrices $A$ and $B$ respectively. When these conditions are satisfied for each of these three matrices, non-negative condition for conditional variance is fulfilled. Log-likelihood function for BEKK-MGARCH model is shown by the following equation:

$$L(\theta) = -(T N / 2) \log (2\pi) - (1 / 2) \sum_{i=1}^{T} (\log |H_i|) + \varepsilon_{t}' H_{t}^{-1} \varepsilon_{t}$$

$\theta$ term in equation (13) stands for all unknown parameters in $\varepsilon_t$ and $H_t$, while $T$ and $N$ stand for sample size and the number of mean equations respectively. In order to obtain maximum likelihood estimates of parameters, Broyden-Fletcher-Goldfarb-Shanno (BFGS) numerical optimization algorithm is used.

4. Empirical Results on the Price Relationship between Spot and Futures Markets

4.1. Johansen Cointegration Test Results

In order to examine the long-run steady relationship between the price series of ISE-30 index futures and ISE-30 index, the series must be tested for stationarity. Logarithmic price series are tested for stationarity using ADF (Augmented Dickey-Fuller) unit root test. Schwarz
The information criterion is used to determine the optimal lag length. For logarithmic price series, Table 1 presents ADF unit root test results for regression model with intercept and also the model with both intercept and trend terms.

<table>
<thead>
<tr>
<th>Series</th>
<th>Level Intercept</th>
<th>Trend and Intercept</th>
<th>First Difference Intercept</th>
<th>Trend and Intercept</th>
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<td>ISE-30 Index</td>
<td>-0.907201</td>
<td>-0.837851</td>
<td>-135.6511**</td>
<td>-135.6526*</td>
</tr>
<tr>
<td>ISE-30 Index Futures</td>
<td>-0.913640</td>
<td>-0.837324</td>
<td>-223.8123**</td>
<td>-223.8130*</td>
</tr>
</tbody>
</table>

* The null hypothesis is rejected at 1% significance level. McKinnon critical values are -3.43, -2.86, -2.57 for the model with intercept and trend -3.96, -3.41, -3.12 for the model with both intercept and trend, at 1%, 5% and 10% significance level respectively.

ADF unit root test results presented in Table 1 reveal that null hypotheses suggesting the series are non-stationary at their levels can not be rejected. The null hypotheses suggesting the series are non-stationary are rejected for the first differenced series, thereby resulting in the acceptance that series are first difference stationary at 1% significance level. This result means that logarithmic price series of both ISE-30 index and ISE-30 index futures are first difference stationary. In other words, this result indicates that both price series are integrated of order 1, denoted by ISE-30 index~I(1) and ISE-30 index futures~I(1).

Johansen Cointegration Test is used to determine whether the price series of ISE-30 index and ISE-30 index futures are cointegrated. In other words the existence of a steady long-run relationship between two series is investigated by the test. Johansen Cointegration Test uses trace ($\lambda_{trace}$) and maximum eigenvalue ($\lambda_{max}$) test statistics to investigate the cointegration relationship between variables. Table 2 presents $\lambda_{trace}$ and $\lambda_{max}$ test results used to determine whether two series are cointegrated or not. Since the test statistics are greater than critical values, the null hypotheses suggesting no cointegration vector ($r = 0$) between the series are rejected at the 5 % significance level, while alternative hypotheses ($r > 0$) are accepted. The null hypotheses of at least one cointegrating vector ($r \leq 1$) between the series are accepted at the 5 % significance level, for the test statistics are smaller than the critical values. This finding is an indication of a steady long-run relationship between ISE-30 index and ISE-30 index futures contracts and of a co-movement of spot and futures markets in the long-run.

<table>
<thead>
<tr>
<th>Series</th>
<th>Test</th>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>Test Statistics</th>
<th>Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISE-30 Index</td>
<td>$\lambda_{trace}$-Test</td>
<td>$r = 0$</td>
<td>$r &gt; 0$</td>
<td>128.52*</td>
<td>15.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$r \leq 1$</td>
<td>$r &gt; 1$</td>
<td>0.82**</td>
<td>3.84</td>
</tr>
<tr>
<td>ISE-30 Index Futures</td>
<td>$\lambda_{max}$-Test</td>
<td>$r = 0$</td>
<td>$r &gt; 0$</td>
<td>127.70*</td>
<td>14.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$r \leq 1$</td>
<td>$r &gt; 1$</td>
<td>0.82**</td>
<td>3.84</td>
</tr>
</tbody>
</table>

* The null hypothesis is rejected at 5 % significance level, while the alternative hypothesis is accepted. ** The null hypothesis is accepted at 5 % significance level, while the alternative hypothesis is rejected. The critical values in the table are Osterwald-Lenum's (1992) critical values.
4.2. Results of the VAR Model

Stoll and Whaley (1990) suggests that some micro-structural differences between spot index and index future contracts might lead to spurious results regarding the direction of lead-lag relationship. For example, not all stocks in the index may continuously be traded in all time intervals. In such a case, spot index price which reflects the average of last trading prices of the stocks in the index may fall behind the current developments in the stock market. Assuming that index futures prices reflect the information immediately, the observed futures prices might lead the observed spot prices. The prices used to calculate returns are trading prices, and trading prices tend to fluctuate randomly within the bid-ask spread. Although the actual returns are serially independent, the random price movements between bid and ask prices in consequent trades can lead to negative serial correlation in the observed returns. In addition, there are time delays in the calculation and reporting of the stock index. With the assumption that new information reaches stock markets and futures markets simultaneously and the price change in the futures market occurs immediately, the time delays in the spot index will result in the tendency of futures returns to lead index returns (Stoll and Whaley, 1990).

Because not all stocks in the index are continuously traded in all time intervals and to eliminate serial correlation effects, Stoll and Whaley (1990) suggest estimating appropriate ARMA models and searching for the relationship between two markets by using the series obtained from the ARMA models. Many researchers like Abhyankar (1998), Turkington and Walsh (1999), Wahab and Lashgari (1993), Kutner and Sweeney (1991), Kang et al. (2006) advocate the same approach in their studies.

Logarithmic price series of ISE-30 index and ISE-30 index futures contracts are transformed into return series by first differencing. Then, the most appropriate ARMA (p,q) model is determined. The use of ARMA models requires series to be stationary. Both return series are stationary (Table 1). ARMA (2,2) is determined to be the most appropriate model structure for ISE-30 index, while ARMA (3,3) turns out to be the model fitting ISE-30 index futures contracts. The residuals of ARMA models are also statistically proven to have neither auto-correlation nor partial auto-correlation.

Because the series obtained from ARMA models are level-stationary, VAR model is used to examine the relationship between the series. Two different VAR models are estimated in which ISE-30 index is one and ISE-30 index futures is the other dependent variable. Schwarz Information Criterion is used to determine the optimal lag length for VAR models. According to Schwarz Information Criterion, optimal lag length for VAR models is determined as eight. The results of the estimated VAR models are presented in Table 3.

In the VAR model where ISE-30 index is the dependent variable, all the coefficients of lagged values of both ISE-30 index and ISE-30 index futures are statistically significant. Therefore, the current changes in the ISE-30 index occurring throughout the day are affected by the lagged values of the index itself, as well as by the lagged price changes of ISE-30 index futures. A similar case is valid in the VAR model where ISE-30 futures is the dependent variable. The current changes in the price of ISE-30 index futures occurring throughout the day are affected by both its own lagged values and the lagged values of ISE-30 index. These findings present evidence that a two-way interaction between spot and futures markets exists.
### Table 3. VAR Model Results

<table>
<thead>
<tr>
<th></th>
<th>Spot</th>
<th>Futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot(-1)</td>
<td>-0.261460***</td>
<td>0.038627***</td>
</tr>
<tr>
<td></td>
<td>[36.6874]</td>
<td>[5.69991]</td>
</tr>
<tr>
<td>Spot(-2)</td>
<td>-0.141998***</td>
<td>-0.001715</td>
</tr>
<tr>
<td></td>
<td>[-19.1290]</td>
<td>[-0.24293]</td>
</tr>
<tr>
<td>Spot(-3)</td>
<td>-0.089785***</td>
<td>-0.008032</td>
</tr>
<tr>
<td></td>
<td>[-11.9970]</td>
<td>[-1.12857]</td>
</tr>
<tr>
<td>Spot(-4)</td>
<td>-0.050753***</td>
<td>0.006953</td>
</tr>
<tr>
<td></td>
<td>[-6.76918]</td>
<td>[0.97516]</td>
</tr>
<tr>
<td>Spot(-5)</td>
<td>-0.038244***</td>
<td>0.010690</td>
</tr>
<tr>
<td></td>
<td>[-5.10337]</td>
<td>[1.50010]</td>
</tr>
<tr>
<td>Spot(-6)</td>
<td>-0.025644***</td>
<td>0.023061***</td>
</tr>
<tr>
<td></td>
<td>[-3.43372]</td>
<td>[3.24731]</td>
</tr>
<tr>
<td>Spot(-7)</td>
<td>-0.015686**</td>
<td>0.030450***</td>
</tr>
<tr>
<td></td>
<td>[-2.12482]</td>
<td>[4.33776]</td>
</tr>
<tr>
<td>Spot(-8)</td>
<td>-0.017977***</td>
<td>0.011117*</td>
</tr>
<tr>
<td></td>
<td>[-2.57613]</td>
<td>[1.67529]</td>
</tr>
</tbody>
</table>

** indicates significance at 10%, 5% and 1% significance levels respectively. Critical $t$ values at 10%, 5% and 1% significance levels are 1.64, 1.96 and 2.33 respectively. The values in brackets are $t$ values of corresponding coefficients.

### 4.3. Granger Causality Test Results

Granger causality test is performed in order to determine whether there is a causality relationship between spot and futures markets and the direction of such a relationship. Granger Causality test results are presented in Table 4. As can be seen in Panel A, the null hypothesis, claiming that the intra-day price changes in ISE-30 index futures do not Granger cause the intra-day price changes in the ISE-30 index is rejected at 1% significance level. Panel B reveals that the null hypothesis, claiming that the intra-day price changes in ISE-30 index future do not Granger cause the intra-day price changes in the ISE-30 index future is rejected at 1% significance level. Rejection of both null hypotheses is an indication of a two-way causality relationship between spot and futures markets. In other words, price changes in the futures market affect price changes in the spot market and vice versa. The absence of a one-way relationship between spot and futures markets means that neither of the markets lead the other one, hence no lead-lag relationship between them. This indicates that neither of the markets react faster to new information and reflect new information in the prices sooner than the other market. In the price formation process, one market is not dominant compared to the other one. Bi-directional causality is also found between logarithmic price series of spot and futures markets, using VECM and Granger causality test to investigate lead-lag relationship between them. However, the results are not reported.

### Table 4. VAR Granger Causality Test Results

<table>
<thead>
<tr>
<th>Panel</th>
<th>H0: ISE-30 index futures does not Granger cause ISE-30 index.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A</td>
<td>Chi-Square: 2186.332* Prob: [0.0000]</td>
</tr>
<tr>
<td>Panel B</td>
<td>Chi-Square: 61.38387* Prob: [0.0000]</td>
</tr>
</tbody>
</table>

* indicates significance at 1% significance level.
5. Empirical Results on the Volatility Relationship between Spot and Futures Markets

The volatility in any market is caused by the change in investor expectations due to new information flows to the market at different points in time regarding company specific and macro-economic factors. Volatility can be thought of as an information source about the reactions and new expectation forms of investors regarding risk and return. In two fundamental studies (French and Roll, 1986; Ross, 1989) on the subject, variance is found to be an important information source on the information flow (So and Tse, 2004; Bose, 2007). Ross (1989) suggests that, in an economy where no arbitrage opportunities exist, price volatility is directly related to the rate of information flow to the market, and that the variance in the rate of price change will be equal to the variance in the rate of information flow. If there is a relationship between spot and futures prices, there should also be a relationship between the volatilities of these markets. Given that volatility is a source of information, if futures markets contain more information and reflect it faster than spot markets then the futures market volatility must lead the spot market volatility.

5.1. The Results of BEKK Multivariate GARCH Model

Understanding the co-movements in returns, as well as modelling return volatility, is a crucial means used for portfolio choice and risk management decision process. The investigation of the volatility in markets and co-movements in volatilities of markets are the fields where multivariate ARCH models (MGARCH) are most frequently used (Öztürk, 2010). In this section of the study the relationship between the volatilities of ISE-30 index futures contracts and ISE-30 index is investigated. Volatility series of the variables need to be obtained for this investigation. Therefore, first volatilities of the variables are modelled using BEKK-MGARCH (1,1) and then volatility series of the variables are obtained from the model. The relationship between volatility series obtained from BEKK-MGARCH (1,1) are examined by VAR and Granger Causality test.

In this study, initially the most appropriate time series model for volatility modelling is searched for. Test results shown in Table 5 are the test results of residual series in the equations in the VAR model for ISE-30 index futures and ISE-30 index series. Ljung-Box Q statistics along with the $Q^2$ statistics and their corresponding p-values for both residual series are presented in the table. Q statistics is used to determine whether residual series are serially correlated, while $Q^2$ statistics is used to find out whether they contain ARCH effect (conditional heteroscedasticity). Q statistics reveal that neither of the series are serially correlated. $Q^2$ statistics of squared residuals indicate the rejection of the null hypothesis of no ARCH effect in residual series, at 1% significance level. The existence of ARCH effect in the residual series requires the use of ARCH (GARCH) type models which take into account ARCH effect in volatility modelling.

Table 5. Test Results of Residual Series

<table>
<thead>
<tr>
<th>Residual Series</th>
<th>$Q_{(4)}$ (m)</th>
<th>$Q_{(8)}$ (m)</th>
<th>$Q_{(12)}$ (m)</th>
<th>$Q^2_{(4)}$ (m)</th>
<th>$Q^2_{(8)}$ (m)</th>
<th>$Q^2_{(12)}$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISE-30 Index</td>
<td>0.0334</td>
<td>1.4853</td>
<td>5.4237</td>
<td>24.078*</td>
<td>27.123*</td>
<td>28.942*</td>
</tr>
<tr>
<td></td>
<td>[1.000]</td>
<td>[0.993]</td>
<td>[0.942]</td>
<td>[0.000]</td>
<td>[0.001]</td>
<td>[0.004]</td>
</tr>
<tr>
<td>ISE-30 Index Futures</td>
<td>0.0002</td>
<td>0.0078</td>
<td>4.6643</td>
<td>42.124*</td>
<td>47.667*</td>
<td>52.666*</td>
</tr>
<tr>
<td></td>
<td>[1.000]</td>
<td>[1.000]</td>
<td>[0.968]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
</tbody>
</table>

*“Q_{(m)}” and “$Q^2_{(m)}$” denote Ljung-Box test statistics for serial correlation of order m, for residual series and squared residual series respectively. Numbers in brackets are p-values of test statistics. * indicates rejection of the null hypothesis of no ARCH effect at 5% significance level.
The estimation results of variance equations in the BEKK-MGARCH model are given in Table 6. The model parameters presented in Table 6 are significant at 1% significance level. Parameter estimation values of the diagonal elements $C(1,1)$ and $C(2,2)$ of coefficient matrix $C$ are positive and statistically significant at 1% significance level. These estimation results indicate that BEKK-MGARCH (1,1), which is the conditional variance of VAR(8)-BEKK-MGARCH (1,1) model constructed for the series, is appropriate. In addition, residual series obtained from the model are not serially correlated. Two variable BEKK-MGARCH (1,1) model turns out to be successful in modelling 5 minute return volatilities of ISE-30 index and ISE-30 index futures contracts.

### Table 6. BEKK-MGARCH(1,1) Component Model Estimation Results

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Transformed Variance Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C(1,1)$</td>
<td>1.13E-08 2.26E-10 50.12772 0.0000</td>
</tr>
<tr>
<td>$C(1,2)$</td>
<td>7.33E-09 9.51E-11 77.01519 0.0000</td>
</tr>
<tr>
<td>$C(2,2)$</td>
<td>7.19E-09 7.22E-11 99.62568 0.0000</td>
</tr>
<tr>
<td>$A(1,1)$</td>
<td>0.071996 0.000390 184.5566 0.0000</td>
</tr>
<tr>
<td>$A(2,2)$</td>
<td>0.070543 0.000227 310.5906 0.0000</td>
</tr>
<tr>
<td>$B(1,1)$</td>
<td>0.996802 3.44E-05 29005.99 0.0000</td>
</tr>
<tr>
<td>$B(2,2)$</td>
<td>0.997070 1.66E-05 59908.10 0.0000</td>
</tr>
</tbody>
</table>

5.2. VAR Model Results

The square root of conditional variance of VAR(8)-BEKK-MGARCH (1,1) model estimated to investigate the relationship between the volatilities of ISE-30 index and ISE-30 index futures contracts is used as the volatility series. To examine the relationship between volatility series, first whether these series are stationary or not is searched for. Schwarz information criterion is used to determine the optimal lag length. ADF unit root test results performed on volatility series are presented in Table 7.

### Table 7. ADF Unit Root Test Results for Volatility Series

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Trend and Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISE-30 Index Volatility Series</td>
<td>-7.100288* -7.152651*</td>
</tr>
<tr>
<td>ISE-30 Index Futures Volatility Series</td>
<td>-5.861245* -5.902698*</td>
</tr>
</tbody>
</table>

* indicates the rejection of the null hypothesis at 1% significance level. MacKinnon critical values are -3.43, -2.86, -2.57 for the model with intercept and -3.96, -3.41 and -3.12 for the model with trend and intercept at 1%, 5% and 10% significance levels respectively.

According to ADF unit root test results, the null hypotheses of non-stationarity of series are rejected for volatility series of both variables at 1% significance level. This result indicates that both ISE-30 index and ISE-30 index futures volatility series are level-stationary.

The dynamic relationship between level-stationary volatility series of ISE-30 index and ISE-30 index futures is examined within the context of VAR model. In order for the model to be estimated, the optimal lag length must be determined initially. The optimal lag length to be used in VAR models are determined as 2 by the Schwarz Information Criterion. Table 8 presents, for volatility series, the estimation results obtained from the VAR model.
The t-statistics of the coefficients belonging to the variables in the model indicate that these coefficients are significant at 1% significance level. The significant coefficients of ISE-30 index volatility variable indicate that ISE-30 index is affected by its own lagged values as well as by the lagged values of ISE-30 index futures volatility variable. Similarly, the significant coefficients of ISE-30 index futures volatility variable show that the volatility of ISE-30 index futures is affected by its own lagged values as well as by the lagged values of ISE-30 index volatility variable. Findings of the VAR model indicate a bi-directional causality between ISE-30 index and ISE-30 index futures contracts.

5.3. Granger Causality Test Results

In order to determine the direction of the relationship between the volatility series of ISE-30 index and ISE-30 index futures estimated by BEKK-MGARCH (1,1) model, Granger causality test is performed. Granger causality test results are presented in Table 9. Panel A reveals that, the null hypothesis that ISE-30 index futures volatility does not Granger cause ISE-30 index volatility is rejected at 1% significance level. Panel B reveals that, the null hypothesis that ISE-30 index volatility does not Granger cause ISE-30 index futures volatility is rejected at 1% significance level. Rejection of both null hypotheses indicates that there is a two-way relationship between the volatilities of spot and futures markets. The change in the volatility of futures markets affects spot market volatility, while the change in the volatility of the spot market affects future market volatility. This finding is consistent with the results of the VAR model.

6. Conclusion

Following the analyses performed we conclude that there is a stable long-run relationship between spot and futures markets, and two markets move together in the long-
run. This conclusion shows that individual and institutional investors investing in the stock markets and portfolio managers can use index futures contracts in order to be protected against price risk.

However, contrary to our expectations, in the context of the relationship between them the futures market turns out not to lead the spot market. The causality relationship between two markets is two-way, hence spot and futures markets have a bilateral interaction in terms of both price and volatility. Therefore, it is found that neither of the markets leads the other one and there is no lead-lag relationship between them. This indicates that neither of the markets reacts faster to new information than the other one, meaning that neither of the markets reflects information in the prices earlier than the other one, and that the information flow is two-way. In other words, neither of the markets is dominant over the other one in the price formation process. The short history of futures and options market in Turkey (founded in 2005 as TurkDEX), the inadequate awareness of investors about how to benefit from this market and low trading volume relative to the spot market in comparison with the developed markets might have influenced such a conclusion. The trading volume in futures market has been continuously increasing. If the trading volume continues to increase, the direction of the relationship between spot and futures markets can change in favor of futures markets.

As far as market efficiency is concerned, a perfect simultaneous relationship between the price changes in two markets is required and the price changes must not be affected by their own and each other’s lagged price changes. According to the analyses performed, there is a two-way relationship between the price changes, and the prices changes are affected by their own and each other’s lagged values. This presents a contradiction with the efficient market hypothesis.

References


